LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2023**

UMT 1501 – ALGEBRA

Date: 06-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION A			
Answer ALL the Questions			
1.	**Answer the following** **(5 x 1 = 5 marks)**		
a)	Define a polynomial in x of n^{th} degree.	K1	CO1
b)	Identify the reciprocal equation. (i). $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0.$ (ii). $x^4 + 5x^3 + 11x^2 - 13x + 6 = 0.$	K1	CO1
c)	Write the value of $\frac{e+e^{-1}}{2}$.	K1	CO1
d)	Define similar matrices.	K1	CO1
e)	List the number of integers less than and prime to 729.	K1	CO1
2.	**Fill in the blanks** **(5 x 1 = 5 marks)**		
a)	The product of the roots of the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ is ________	K1	CO1
b)	The number of real roots in the equation $x^5 - 6x^2 - 4x + 5 = 0$ is ______.	K1	CO1
c)	The general term in the expansion of $(x + a)^n$ is ___________.	K1	CO1
d)	The eigen values of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ is ________________.	K1	CO1
e)	The product of r consecutive integers is divisible by __________.	K1	CO1
3.	**Choose the correct answer for the following** **(5 x 1 = 5 marks)**		
a)	The sum of the roots of the equation $81x^3 - 18x^2 - 36x + 8 = 0$ is i) $\frac{-2}{9}$ ii) $\frac{2}{9}$ iii) $\frac{-2}{8}$ iv) $\frac{2}{8}$	K2	CO1
b)	The number of negative roots of the equation $4x^3 - 21x^2 + 18x + 20 = 0$ is i) 1 ii) 2 iii) 3 iv) none	K2	CO1
c)	The expansion of $\frac{e^x - e^{-x}}{2}$ is i). $1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots \ldots \ldots . \infty$ ii). $1 - \frac{x}{1!} + \frac{x^2}{2!} + \cdots \ldots \ldots . \infty$ iii). $1 + \frac{x^2}{2!} + \frac{x^4}{4!} \ldots \ldots \ldots . \infty$ iv). $x + \frac{x^3}{3!} + \frac{x^5}{5!} \ldots \ldots \ldots . \infty$	K2	CO1
d)	The product of the eigen value of the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is i) 6 ii) -6 iii) 8 iv) -8	K2	CO1
e)	The number of divisors of 360 is i) 20 ii) 24 iii) 22 iv) 18	K2	CO1
4.	**Say TRUE or FALSE** **(5 x 1 = 5 marks)**		
a)	If $f(a)$ and $f(b)$ are of like unlike signs, an odd number of roots of $f(x)$ lies between a and b.	K2	CO1

b)	No equations can have a greater number of negative roots then there are changes of sign in the terms of the polynomial $f(-x)$.	K2	CO1
c)	The number of terms in the binomial expansion of $(x+a)^n$ is $n+2$.	K2	CO1
d)	If A and B are similar matrices then they do not have the same characteristic equation.	K2	CO1
e)	If $a \equiv b(mod\ m)$, then $a^n \equiv b^n(mod\ m)$.	K2	CO1
	SECTION B		
Answer any TWO		**(2 x 10 = 20 marks)**	
5	Show that the roots of the equation $x^3+px^2+qx+r=0$ are in arithmetical progression if $2p^3-9pq+27r=0$.Show that the above condition is satisfied by the equation $x^3-6x^2+13x-10=0$.	K3	CO2
6	Determine the transformed equation by diminishing the roots of the equation $x^4-5x^3+7x^2-4x+5=0$ by 2.	K3	CO2
7	Interpret the value of the sum the series $1+\frac{1+3}{2!}+\frac{1+3+3^2}{3!}+\frac{1+3+3^2+3^3}{4!}+$……..to ∞.	K3	CO2
8	Computer the inverse of the matrix $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ after predicting the characteristics equation.	K3	CO2
	SECTION C		
Answer any TWO		**(2 x 10 = 20 marks)**	
9	Determine the roots of the equation $x^3-9x^2+108=0$ by using cardon's method.	K4	CO3
10	Resolve into partial fraction $\frac{x^2-10x+13}{(x-1)(x^2-5x+6)}$.	K4	CO3
11	Verify Cayley Hamilton theorem for the matrix $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$	K4	CO3
12	Examine whether $13^{2n+1}+9^{2n+1}$ is divisible by 22.	K4	CO3
	SECTION D		
Answer any ONE		**(1 x 20 = 20 marks)**	
13	a) Predict all the roots of the equation $6x^6-35x^5+56x^4-56x^2+35x-6=0$. (10 marks)	K5	CO4
	b) Estimate a positive root of the equation $x^3-3x+1=0$ by Horner's method which lies between 1 and 2, correct to two decimal places. (10 marks)	K5	CO4
14	a) Determine the eigen values and eigen vectors of the matrix $A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$	K5	CO4
	b) i). Estimate the remainder when 2^{46} is divided by 47.	K5	CO4

		ii). Justify that $(\sum x)^3 - 3\sum x^3$ is divisible by 108 only when x, y, z are three consecutive integers. (5+ 5)		
SECTION E				
Answer any ONE		**(1 x 20 = 20 marks)**		
15	a)	Solve the equation $x^4 + 20x^3 - 143x^2 + 430x + 462 = 0$ by removing the second term. (10 marks)	K6	CO5
	b)	If $\propto, \beta, \gamma$ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $(\alpha^2 + 1)(\beta^2 + 1)(\gamma^2 + 1)$. (10 marks)	K6	CO5
16		Diagonalise the matrix $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$.	K6	CO5

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